The Roper and Radiative Decay of Pentaquarks

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Abstract

Identifying the Roper $N(1440)$ and $N(1710)$ as the pentaquark octet and anti-decuplet, respectively, we analyze their main decay modes in the diquark picture. The ratio of the partial decay widths is largely consistent with the nearly ideal mixing of the Jaffe-Wilczek diquark model, which then allows to predict the width of the radiative decay of $N(1440)$, $\Gamma_{10}(N \to p\gamma) = 1/4\,\Gamma_{12}(N \to n\gamma) = 0.25 - 0.31$ MeV. We then show that the three-body radiative decay of the pentaquark anti-decuplet is quite enhanced due to its mixing with the pentaquark octet. We find for the $J^P = \frac{1}{2}^+$ pentaquark anti-decuplet $\Gamma(\Theta^+ \to K^+\, n\, \gamma) = 0.034 \sim 0.041$ MeV. The diquark picture of the pentaquark predicts $\Gamma(\Theta^+ \to K^+\, n\, \gamma) = 4\,\Gamma(\Theta^+ \to K^0\, p\, \gamma)$. Finally we show that the difference in the Θ^+ mass in the $K^0 p$ and $K^+ n$ decay channels may be accounted for by the missing photons in the radiative decay.

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Since the discovery of a narrow state of exotic baryons in the photon-nucleon scattering at LEPS [1] as predicted by chiral soliton models [2], several models have been proposed for exotic baryons [3]. However, in spite of the active theoretical study on exotic baryons, the current experimental situation of pentaquarks [4] is quite confusing, as some of the subsequent experiments have not seen them.

In this letter, we analyze the decay modes of $N(1440)$ and $N(1710)$ to show that they are consistent with the Jaffe-Wilczek (JW) diquark model [5]. We then point out that the three-body radiative decay of the pentaquark anti-decuplet can be quite enhanced due to the near degeneracy between the octet and the anti-decuplet and also the diquark nature of pentaquarks. Since the radiative decay amplitude is proportional to the electric charge of the anti-quark bound in the pentaquark, the diquark model predicts $\Gamma(\Theta^+ \to\ K^+ n\,\gamma) = 4\,\Gamma(\Theta^+ \to\ K^0 p\,\gamma)$, which will be a clear signal for the diquark models.

In the JW model, the exotic baryons are bound states of two scalar diquarks and one anti-quark, forming the multiplets of low dimensions, $\overline{10}$ and 8, of the SU(3) flavor symmetry. The degeneracy of $\overline{10}$ and 8 is lifted by a nearly ideal mixing between them. It was soon confirmed by NA49 experiments [6], which discovered $S = -2$ pentaquarks of mass 1860 MeV, not much different from the JW prediction. The decay widths of pentaquarks are also explicitly calculated in the diquark effective theory based on the JW model to find they are naturally small, a few MeV or less, since they decay through tunnelling the potential barrier between two diquarks [7].

In the JW model, the lightest member of the pentaquark octet is identified, after mixing, as the Roper state $N(1440)$. Its orthogonal state in the anti-decuplet is identified as $N(1710)$. Though the assignment fits well in the mass formula, it is argued in [8, 9] that such an identification leads to a gross violation of SU(3) in partial decay widths, unseen in other hadronic decays. However, we show that the partial decay widths of $N(1440)$ and $N(1710)$ are consistent with the JW model, because the gross violation is due to the fact that the exotic baryons decay by tunneling, whose amplitude depends exponentially on the SU(3) violating terms. Another concern [10] was that the decay branching fraction for $\Delta(1600) \to N(1440)\,\pi$ is not suppressed at all, $\Gamma_1/\Gamma = 10 - 25\%$ [11]. But, since the exotic baryon production is suppressed not by tunneling but only by the production amplitude of diquarks, whose strength is nothing but the Yukawa coupling of the diquark to two quarks, $g \approx 1.7$ [7], the observed branching ratio does not necessarily contradict with the diquark picture.

Finally, we show that a three-body radiative decay is quite enhanced and contributes significantly to the decay of the pentaquark anti-decuplet. In the JW model, due to the mixing, the diquarks inside the octet are closer to each other than in the anti-decuplet, making tunnelling easier for the octet. Furthermore, since the octet and the anti-decuplet are almost degenerate, the virtual octet in the three-body decay of the anti-decuplet is near on-shell and makes the three-body decay more efficient.

In the ideal mixing, the quark content of the $S = 0$ component of the pentaquark octet is $\mathcal{N}_1 = |\bar{q}, \varphi_{ud}, \varphi_{ud}\rangle$, where $\bar{q}$ is the light anti-quark and φ_{ud} is the diquark made of u and d, while the $S = 0$ component of the pentaquark anti-decuplet is $\mathcal{N}_2 = |\bar{s}, \varphi_{qs}, \varphi_{ud}\rangle$, where the φ_{qs} diquark is made of the light quark q and the strange quark, s.

The dominant decay modes of $\mathcal{N}_1$ and $\mathcal{N}_2$ are shown in the diquark picture in Fig. 1. From the viewpoint of the chiral perturbation theory, the processes shown in Fig. 1 (a)

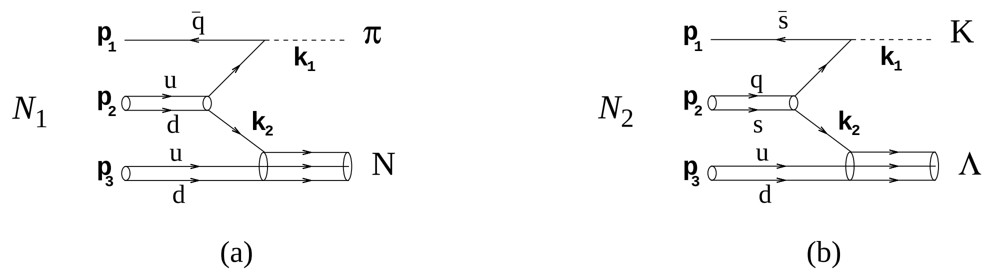

FIG. 1: The dominant decay modes of excited nucleons: (a) $\mathcal{N}_1 \to N\pi$ (b) $\mathcal{N}_2 \to \Lambda K$. The breaking of diquark occurs through tunneling.

and (b) should not differ by more than 30%, the typical size of SU(3) breaking terms. However, the experimental data differs by an order of magnitude, if $\mathcal{N}_1$ and $\mathcal{N}_2$ are identified as $N(1440)$ and $N(1710)$, respectively: $\Gamma_a\,(N(1440) \to N\,\pi) = 210 - 245$ MeV and $\Gamma_b\,(N(1710) \to \Lambda\,K) = 5 - 25$ MeV. This may seem to contradict with the diquark picture. But, a gross difference in the decay widths of two processes is expected, since the SU(3) violating terms appear in the exponent of the tunneling amplitude in the diquark picture.

The decay widths are given respectively as

$$\Gamma_a = \frac{g^2_{\pi N \mathcal{N}_1}}{4\pi\, f_\pi^2} \frac{|\vec{k}_1| \left(M_{N(1440)} + m_N\right)^2}{M^2_{N(1440)}} \left[\left(M_{N(1440)} - m_N\right)^2 - m_\pi^2\right] \tag{1}$$

$$\Gamma_b = \frac{g^2_{K \Lambda \mathcal{N}_2}}{4\pi\, f_K^2} \frac{|\vec{k}_1'| \left(M_{N(1710)} + m_\Lambda\right)^2}{M^2_{N(1710)}} \left[\left(M_{N(1710)} - m_\Lambda\right)^2 - m_K^2\right], \tag{2}$$

where the outgoing meson momenta are $|\vec{k}_1| = 396$ MeV and $|\vec{k}_1'| = 268$ MeV, respectively. From the partial widths by the Particle Data Group (PDG), we get $g_{\pi N \mathcal{N}_1} = \pm\,(0.3 \sim 0.33)$ and $g_{K\Lambda\mathcal{N}_2} = \pm\,(0.10 \sim 0.23)$.

The amplitude for exotic baryons to decay into normal baryons is quite suppressed due to the tunneling barrier for the diquarks [7]. Therefore, the couplings in the decay of $\mathcal{N}_1$ and $\mathcal{N}_2$ have to be proportional to their tunneling amplitudes. The WKB approximation for the tunneling amplitude for $\mathcal{N}_1$ gives $e^{-S_0(\mathcal{N}_1)} \simeq e^{-\Delta E\, r_0} = 0.26$, where the potential barrier is approximately the binding energy of quarks in the scalar diquark, $\Delta E \simeq 270$ MeV, and $r_0 \simeq (201\ \text{MeV})^{-1}$ is the average distance between two diquarks of $\mathcal{N}_1$ in P-wave. The distance is estimated from the mass formula for the naive diquark model

$$M_{\mathcal{N}_1} = 2\, M_{ud} + m_q + \frac{1}{M_{ud}\, r_0^2}, \tag{3}$$

where $M_{ud} \simeq 450$ MeV is the diquark φ_{ud} mass [7] and $m_q = 360$ MeV is the constituent mass of light quarks. Similarly, the average distance between two diquarks φ_{qs} and φ_{ud} of $\mathcal{N}_2$ in P-wave is estimated to be $r_1 \simeq (145\ \text{MeV})^{-1}$ and the $\mathcal{N}_2$ tunneling amplitude $e^{-S_0(\mathcal{N}_2)} = 0.16$. Indeed, we find that the ratio of the tunneling amplitudes is close to the ratio of the couplings,

$$\frac{e^{-S_0(\mathcal{N}_1)}}{e^{-S_0(\mathcal{N}_2)}} \simeq 1.7, \quad \left|\frac{g_{\pi N \mathcal{N}_1}}{g_{K\Lambda\mathcal{N}_2}}\right| \simeq 1.3 - 3.3 \tag{4}$$

It seems that most partial decay modes of $N(1710)$ are largely consistent with the JW model. However, one of the dominant decay modes, $N(1710) \to N\pi$, does not fit in the JW model. The decay width is given in the chiral perturbation theory as $\Gamma(N(1710) \to N\pi) = g_{\pi N\mathcal{N}_2}^2 7422$ MeV, where $g_{\pi N \mathcal{N}_2}$ is the coupling of $\mathcal{N}_2$ to $\pi\, N$. In the ideal mixing, $\mathcal{N}_2 = |\bar{s}, \varphi_{sq}, \varphi_{ud}\rangle$ does not decay into $N\pi$ at the leading order [7, 12]. However, the decay width is estimated to be $10 - 20$ MeV by PDG. This can be resolved if the mixing between the anti-decuplet and the octet is not exactly ideal but nearly ideal [10]; $\theta = \cos^{-1}\sqrt{2/3} + \delta$. Then $\mathcal{N}_2 \simeq |\bar{s}, \varphi_{sq}, \varphi_{ud}\rangle + \delta\, |\bar{q}, \varphi_{ud}, \varphi_{ud}\rangle$ and the partial decay width becomes

$$\Gamma(N(1710) \to\ N\pi) \approx \delta^2\, \Gamma(N(1440) \to\ N\pi), \tag{5}$$

which gives $\delta = \pm\,|g_{\pi N\mathcal{N}_2}/g_{\pi N \mathcal{N}_1}| \approx \pm\,0.26$ or the mixing angle $\theta \approx 20°$.

We first examine the radiative decay of the Roper $N(1440)$. Assuming it is the mixed pentaquark $\mathcal{N}_1$, we have drawn the diagram for the radiative decay in Fig. 2 (a), neglecting

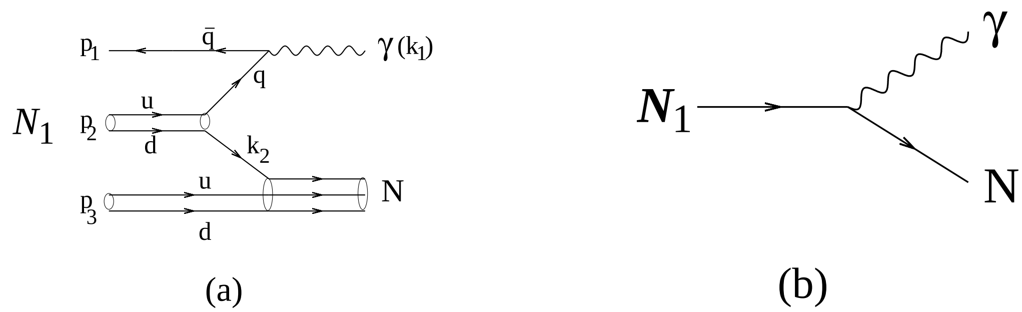

FIG. 2: The radiative decay of the Roper N(1440)

the angle δ. The leading interaction for the radiative decay of the pentaquark octet in the chiral perturbation theory (Fig. 2. b) is given as

$$\mathcal{L}^{(1)}_{\mathcal{O}B\gamma} = e_* \bar{B} \not{A} Q_e \mathcal{O} + \text{h.c.} \tag{6}$$

where B is the normal baryon octet, A_μ is the photon field, and Q_e is the electric charge matrix acting on the anti-quark bound in the pentaquark octet, $\mathcal{O}$. The effective electric charge e_* is proportional to the tunneling amplitude, $e^{-S_0(\mathcal{N}_1)}$, since the decay occurs by annihilation of a quark bound in a diquark after the other quark in the diquark tunnels to another diquark.

For the radiative decay we get, after summing over the photon polarization,

$$\Gamma\left(\mathcal{N}_1 \to N\gamma\right) = Q_e^2 \frac{e_*^2}{2\pi} \frac{M_{\mathcal{N}_1}^2 - m_N^2}{M_{\mathcal{N}_1}^3} \times \left[2M_{\mathcal{N}_1} m_N - \left(M_{\mathcal{N}_1} - m_N\right)^2\right] = e_*^2 Q_e^2 \, 156 \text{ MeV}, \tag{7}$$

where Q_e is the electric charge of the anti-quark in the pentaquark octet. Since the decay amplitude is proportional to the electric charge of the anti-quark we get at the leading order $\Gamma(\mathcal{N}_1 \to n\gamma) = 4\,\Gamma(\mathcal{N}_1 \to p\gamma)$.

Comparing the decay processes $\mathcal{N}_1 \to N\pi$ (Fig. 1. a) and $\mathcal{N}_1 \to N\gamma$ (Fig. 2. a), we obtain $e_*\, g_A = e\, g_{\pi N \mathcal{N}_1}$, where $g_A \simeq 0.75$ is the quark axial coupling in the quark model. Using the coupling obtained from the decay width for $N(1440) \to N\pi$, we get $e_* = 0.4 - 0.44\ e$. Then, the partial radiative decay width becomes $\Gamma\left(\mathcal{N}_1 \to p\gamma\right) = 0.25 \sim 0.31$ MeV, which is about 2 times larger that the estimate made by PDG [13].

Finally, we consider the three-body radiative decay of pentaquarks. The diagram for $\Theta^+ \to KN$ and the dominant diagram [14] for $\Theta^+ \to KN\gamma$ in the diquark effective theory are shown in Fig. 3. The same processes in the chiral perturbation theory are drawn in Fig. 4. The exotic baryons in the anti-decuplet are coupled to the octet exotics by the

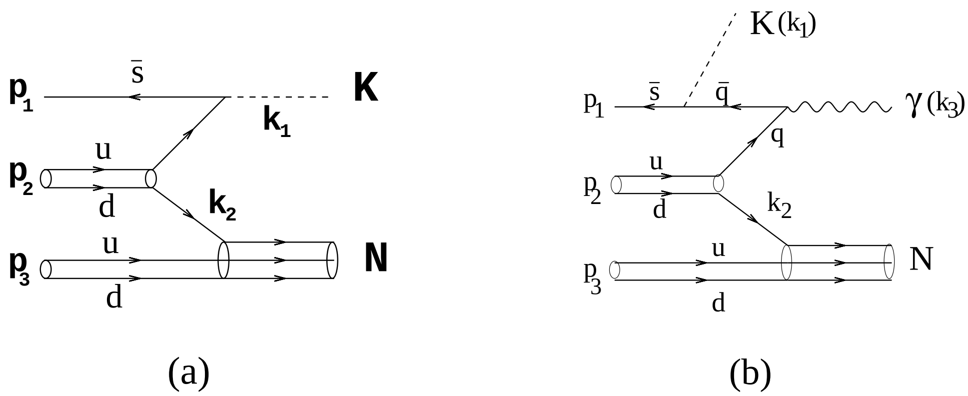

FIG. 3: The decay modes of Θ^+: (a) two-body decay and (b) three-body radiative decay.

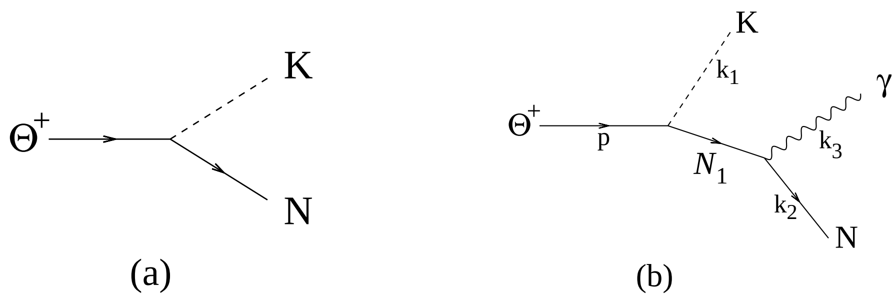

FIG. 4: The decay processes of Θ^+ in chiral perturbation theory

meson octet, as well as to the ground state baryon octet. The leading interaction terms for the decay of the pentaquark anti-decuplet and the pentaquark octet are given in the SU(3) limit as

$$\mathcal{L}^{(1)} = \mathcal{C}_{\mathcal{P}B}\bar{\mathcal{P}}\gamma_5 \not{a} B + g_{\mathcal{P}\mathcal{O}}\bar{\mathcal{P}}\gamma_5 \not{a}\mathcal{O} + \mathcal{D}_{\mathcal{O}B}\,\mathrm{Tr}\left(\bar{\mathcal{O}}\gamma_5 \{\not{a}, B\}\right) + \mathcal{F}_{\mathcal{O}B}\,\mathrm{Tr}\left(\bar{\mathcal{O}}\gamma_5 [\not{a}, B]\right) + \text{h.c.} \quad (8)$$

where $\mathcal{P}$ and $\mathcal{O}$ are the anti-decuplet and octet pentaquarks, respectively, and $a_\mu = i/2(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger)$ is the axial current of the meson octet, $\xi = \exp\left(i\pi_a T_a/f\right)$. ($T_a$'s are the SU(3) generators.)

The couplings $\mathcal{C}_{\mathcal{P}\mathcal{B}}, \mathcal{D}_{\mathcal{O}B}, \mathcal{F}_{\mathcal{O}B}$ are proportional to the axial coupling g_A, the Yukawa coupling between the diquark and quarks, g, and the tunneling amplitude e^{-S_0} [7]. The pentaquark octet will decay into the ground state baryon octet and other particles. But, only the radiative decay is allowed for the virtual octet in the three-body decay of the anti-decuplet pentaquarks because the allowed phase space is below the threshold of any hadronic decays for the intermediate pentaquark octet.

The differential cross-section for the three-body decay $\Theta^+ \to K\,N\gamma$ (Fig. 4. b) is given at tree-level as

$$\frac{\mathrm{d}\Gamma\left(\Theta^+ \to K\,N\gamma\right)}{\mathrm{d}|\vec{k}_3|\,\mathrm{d}\cos\theta_\gamma} = \frac{g^2_{\Theta^+\mathcal{N}_1K}\,e_*^2\,Q_e^2}{128\pi^3}\frac{m_K^3}{M_{\Theta^+}\,f_K^2} \times F(|\vec{k}_3|, \cos\theta_\gamma\,;\,M_{\Theta^+},\,M_{\mathcal{N}_1},\,m_N,\,m_K), \quad (9)$$

where m_K and $f_K \simeq 114$ MeV are the mass and the decay constant of kaons, respectively. The photon energy is $|\vec{k}_3|$ and θ_γ is the angle between the kaon momentum and the photon momentum. The function $F(|\vec{k}_3|, \cos\theta_\gamma\,;M_i)$ has a peak when two momenta are opposite. (See Fig. 5.)

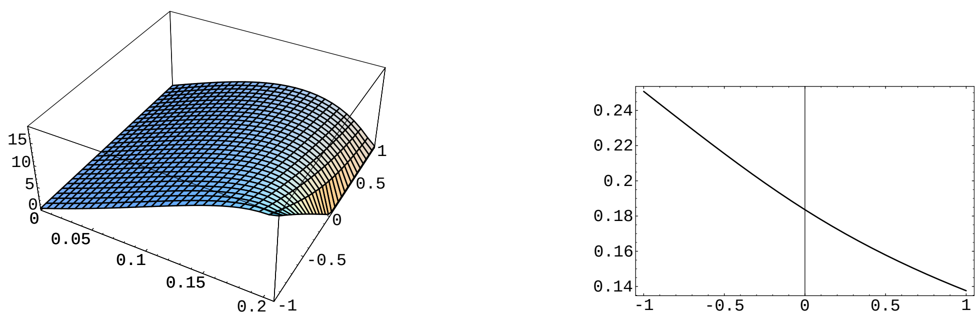

FIG. 5: In the left the differential decay width for $\Theta^+ \to K^+\,n\gamma$ is drawn as a function of photon energy (in unit of m_K) and $\cos\theta_\gamma$. In the right, $\mathrm{d}\Gamma/(\mathrm{d}\cos\theta_\gamma)$ is drawn in 0.1 MeV as $\cos\theta_\gamma$ varies.

Integrating over the angle θ_γ, we get the radiative decay width of the pentaquark Θ^+

$$\Gamma\left(\Theta^+ \to K^+ n\gamma\right) = 4\Gamma\left(\Theta^+ \to K^0\,p\gamma\right) \simeq 1.38\frac{e_*^2\,g^2_{\Theta^+\mathcal{N}_1K}}{256\pi^3}\frac{m_K^4}{M_{\Theta^+}\,f_K^2} \simeq 0.034 \sim 0.041\,\mathrm{MeV}, (10)$$

where we use $e_* = 0.4 \sim 0.44\,e$ and $g_{\Theta^+\mathcal{N}_1K} \simeq 2\sqrt{2}\,g_A = 2.12$, the naive quark model value. In the SU(3) limit the Θ^+ coupling to the octet meson and the ground state baryon is related to the physical couplings of $N(1440)$ and $N(1710)$, which are mixed states of $\left|\overline{10}\right\rangle$ and $|8\rangle$: $\mathcal{C}_{\mathcal{P}B} = \sqrt{2}\,g_{\pi N\mathcal{N}_1}\sin\theta - \sqrt{2}\,g_{\pi N\mathcal{N}_2}\cos\theta$. Using the PDG values for the couplings (of the same sign) and $\theta \simeq 20°$, we find $|\mathcal{C}_{\mathcal{P}B}| = 0.031 \sim 0.056$. Since the two-body decay width of Θ^+ in the chiral perturbation theory [15, 16] is given as $\Gamma(\Theta^+ \to K\,N) = \mathcal{C}^2_{\mathcal{P}B}\,495$ MeV, we get $\Gamma(\Theta^+ \to K^+\,n) = \Gamma(\Theta^+ \to K^0\,p) = 0.24 \sim 0.78$ MeV. Therefore, our analysis shows that the three-body radiative decay contributes quite significantly to the decay of Θ^+: $4.6 \sim 13.5\,\%$ in the $K^+\,n$ channel and $1.2 \sim 3.8\,\%$ in the $K^0\,p$ channel. The slight violation of the isospin symmetry in the decay of Θ^+ is expected in the diquark picture.

We note that the systematic difference (of about 12 MeV) in the measurement of the Θ^+ mass between $K^0\,p$ and $K^+\,n$, noted in [17], may be due to the missing photons in the radiative decay, $\Theta^+ \to K^0\,p\,\gamma$ [18]. In the $K^0\,p$ decay channel, the Θ^+ mass is reconstructed from the invariant mass, M_{K^0p}, of K^0 and p, while it is measured from the missing mass in the K^+n decay process. But, when there is a missing photon of momentum $\vec{k}_3$ in the $K^0\,p$ decay channel of Θ^+, the invariant mass is given in the rest frame of Θ^+ as $M^2_{K^0p} = M^2_{\Theta^+} - 2\,M_{\Theta^+}\,|\vec{k}_3|$. The probability of finding the photon of energy in $|\vec{k}_3| \sim |\vec{k}_3| + \mathrm{d}|\vec{k}_3|$ in the $K^0\,p$ channel is

$$\frac{1}{\Gamma_{\mathrm{tot}}}\left(\frac{\mathrm{d}\Gamma_{3\mathrm{body}}}{\mathrm{d}|\vec{k}_3|}\right)\mathrm{d}|\vec{k}_3|. \tag{11}$$

where $\Gamma_{\mathrm{tot}} = \Gamma(\Theta^+ \to K^0\,p) + \Gamma(\Theta^+ \to K^0\,p\gamma)$ and $\mathrm{d}\Gamma_{3\mathrm{body}}$ is the differential three-body radiative decay width in Eq. (9), after integrating over θ_γ. Averaging over the photon energy distribution, we get

$$M^2_{K^0p} = M^2_{\Theta^+} - 0.26\,M_{\Theta^+}\,m_K\,\frac{\Gamma_{3\mathrm{body}}}{\Gamma_{\mathrm{tot}}}. \tag{12}$$

Therefore, we find $M_{K^0p} \simeq 1530{-}1528$ MeV for $M_{\Theta^+} = 1540$ MeV if $\Gamma_{3\mathrm{body}} \simeq 0.15{-}0.19\,\Gamma_{\mathrm{tot}}$, which is about $5-7$ times larger than our estimate on the radiative decay width. The mass difference is due to missing photons, if the coupling $g_{\Theta^+\mathcal{N}_1K}$ is about 3 times bigger than the naive quark model value or $\mathcal{C}_{\mathcal{P}B}$ is about 1/3 of our estimate.

In conclusion, we analyze the decay modes of the Roper $N(1440)$ and $N(1710)$ in the Jaffe-Wilczek diquark model, provided that they are the mixed states of the pentaquark octet and anti-decuplet. We find that the experimental data on the partial decay widths is consistent with the diquark model, as the phenomenological couplings for the decay modes are proportional to the tunneling amplitude of the diquark barrier. We then predict the radiative decay of $N(1440)$ to be $\Gamma_{10}(N \to p\gamma) = \Gamma_{12}(N \to n\gamma)/4 = 0.25 - 0.31$ MeV. The ratio is the unique prediction of the pentaquark feature of $N(1440)$ in the diquark model.

Finally, the three-body radiative decay of pentaquarks is shown to be quite enhanced in the Jaffe-Wilczek diquark model because of the near degeneracy of the pentaquark anti-decuplet and octet and the larger tunneling amplitude for the octet. From the experimental data on $N(1440)$ and $N(1710)$, we calculate the radiative decay width of Θ^+ in chiral perturbation theory to find about 0.04 MeV in $K^+\,n\gamma$ channel and 0.01 MeV in $K^0\,p\gamma$ channel. It is also shown that the mass difference of Θ^+ in the K^0p and K^+n decay channels may be accounted for by the missing photons in the radiative decay.

The three-body radiative decay is quite significant in the decay of Θ^+, especially in the forward decay, since it has a peak when the outgoing kaon and nucleon are collinear. Our analysis suggests that one should take into account the radiative decay in search of the pentaquark decay.

Acknowledgments

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[1] T. Nakano *et al.* [LEPS Collaboration], Phys. Rev. Lett. **91** (2003) 012002 [arXiv:hep-ex/0301020].

[2] D. Diakonov, V. Petrov and M. V. Polyakov, Z. Phys. A **359**, 305 (1997) [arXiv:hep-ph/9703373]; M. Praszalowicz, in Proceedings, "Skyrmions And Anomalies", M. Jezabek and M. Praszalowicz, eds., World Scientific (1987), p. 112; A. V. Manohar, Nucl. Phys. B **248**, 19 (1984); M. Chemtob, Nucl. Phys. B **256**, 600 (1985).

[3] For recent reviews, see R. L. Jaffe, arXiv:hep-ph/0409065; M. Karliner and H. J. Lipkin, arXiv:hep-ph/0307243; M. Praszalowicz, arXiv:hep-ph/0410241.

[4] For a recent overview on the pentaquarks experiments, see G. Trilling in [11].

[5] R. L. Jaffe and F. Wilczek, Phys. Rev. Lett. **91** (2003) 232003 [arXiv:hep-ph/0307341].

[6] C. Alt *et al.* [NA49 Collaboration], Phys. Rev. Lett. **92**, 042003 (2004) [arXiv:hep-ex/0310014].

[7] D. K. Hong, Y. J. Sohn and I. Zahed, Phys. Lett. B **596**, 191 (2004) [arXiv:hep-ph/0403205].

[8] T. D. Cohen, Phys. Rev. D **70**, 074023 (2004) [arXiv:hep-ph/0402056].

[9] K. Goeke, H. C. Kim, M. Praszalowicz and G. S. Yang, arXiv:hep-ph/0411195.

[10] V. Mohta, arXiv:hep-ph/0411247.

[11] S. Eidelman *et al.* [Particle Data Group Collaboration], Phys. Lett. B **592**, 1 (2004).

[12] D. K. Hong and C. j. Song, arXiv:hep-ph/0407274.

[13] In a recent experiment at JLab, they found strong longitudinal response to $N(1440)$ in $\gamma^{*}+p \rightarrow N(1440)$ [19].

[14] There are other tree diagrams for the radiative decay, but they are all suppressed, since they do not have $\mathcal{N}_1$ or have more powers of the electric charge.

[15] T. Mehen and C. Schat, Phys. Lett. B **588**, 67 (2004) [arXiv:hep-ph/0401107].

[16] P. Ko, J. Lee, T. Lee and J. h. Park, arXiv:hep-ph/0312147;

[17] P. Rossi [CLAS Collaboration], arXiv:hep-ex/0409057; K. Hicks, arXiv:hep-ex/0412048.

[18] The author thanks K. Hicks for discussions on this.

[19] I. G. Aznauryan, V. D. Burkert, H. Egiyan, K. Joo, R. Minehart and L. C. Smith, arXiv:nucl-th/0407021.